EXPERIMENTAL INVESTIGATION OF GASDYNAMIC PROCESSES AT SUDDEN START-UP OF A SUPERSONIC NOZZLE

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A sudden start-up of a supersonic nozzle is encountered quite often in cases where the pressure at the nozzle entrance increases in a stepwise manner, particularly in investigations of pulsed gasdynamic lasers, when a reflecting nozzle located at the end of a shock tube is used [1]. The gasdynamic process in the start-up of such nozzles has been investigated for quite some time, but not as yet in sufficient detail [2-10]. This paper examines the basic wave structure which occurs at start-up, in particular the system of initial and reflected waves arising in the expanding part of the nozzle. In references [2-4, 11] numerical calculations were made for some conditions of the start-up process in supersonic nozzles. It was established that for large pressure drops in the nozzle the start-up time is determined by the passage of an unsteady expansion wave through the nozzle, and for small pressure drops by the passage of the reflected wave through the nozzle. In [5-7] a comparison was made of the velocity of the waves with steady-state theory, described in detail in [8]. In [9, 10] the wave structure of the process was investigated fully and the velocities of the wave system were compared with calculations using steady-state theory and the Chisnell theory [12]. Most papers investigated conditions for which an unseparated steady-state flow inside a nozzle is obtained. The objective of the present paper is to improve on the wave structure of the flow and to investigate the parameters over a wider range, in order to also study start-up conditions where the flow separates, where jets form within the nozzle, and where a one-dimensional flow picture is not sufficient.

Experiments were carried out in a single-diaphragm shock tube of square section 72 × 72 mm, which had a two-dimensional nozzle at the end of its low-pressure chamber. The side walls of the nozzle were made of optical glass. Nitrogen and air at initial pressure p_0 = 10, 20, 30, 50, and 150 torr were investigated. The Mach number M_0 of the primary shock wave was varied from 2 to 6. The parameters of the nozzles used are shown in Table 1, where α_0 is the half-opening angle of the nozzle, h is the half-height of the nozzle at the throat, H is the half-height of the nozzle exit, r is the radius of curvature of the nozzle throat, and M_n is the Mach number for which the nozzle was designed.

In the experiments single-spark schlieren photographs (shadowgraph photographs) were taken of the nozzle start-up. The duration of the pulsed spark discharge in air, the light source, was about 0.1 μsec. Continuous photographs of the process with time were also obtained in the from of x, t diagrams. For this a type ZhFR-1 device was used in conjunction with a type IAB-451 instrument. In this case the light source was a type IFK-50 pulsed lamp, giving a light pulse duration on the order of 400 μsec. To obtain the sweep the slit was set up in two ways: along the nozzle axis and along a generator. In the first case the photographic sweep allowed us to determine the coordinates and the velocities of the primary, reflected, start-up and transformed shock wave and the contact surface. In the second case we could determine the coordinates and the velocity of a point of flow separation and reattachment.

The system of waves arising during start-up is shown in Fig. 1 [schlieren photographs of the initial stage of start-up of nozzles with half-angles at the vertex of 5, 15 (model 3), and 30°; the gas is nitrogen, p_0 = 30 torr, and M_0 = 2.36 (5°), 2.40 (15°), 2.20 (30°).

The start-up process proceeds as follows: The primary shock wave decomposes into the reflected shock 4 and the wave passing into the throat, i.e., the start-up wave, which is more intense than the original wave. During subsequent motion along the expanding nozzle the start-up wave 1 attenuates, and a contact surface 2 forms behind it. This separates the gas discharging from the nozzle throat from the gas which filled the nozzle earlier and has been compressed by the start-up wave. The unsteady expansion wave created is carried down-

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TABLE 1

Model number	Type of model	α_0, deg	h, mm	H, mm	r, mm	M_n
1	Nozzle	5	2	11	5	3,2
2	"	15	1	18	4	4,8
3	"	15	3	27	5	3,8
4	"	30	3	36	5	4,2
5	Half-nozzle	15	8	38	25	3,1

stream along the nozzle. Because the pressure of gas which is expanded in the nozzle is less than that behind the starting shock, another shock wave 3 is formed, reflected upstream. This wave interacts with the boundary layer on the nozzle walls, and consequently it has a λ-shaped form near the walls. Figure 1 shows the influence of the nozzle opening angle on the shock configuration. Figure 1a shows a schlieren photograph of the start-up of a nozzle with half-angle of 5°. One can see the starting shock, the contact surface, and the reflected wave which is convex in shape because of interaction with the boundary layer on the nozzle wall. Figure 1b (nozzle with half-angle 15°), in contrast with Fig. 1a, has a reflected wave which is almost planar, while the starting shock has a nearly cylindrical shape. One can easily see the reflected wave and the contact surfaces arising at successive Mach reflections during formation of the reflected wave. These tangential surfaces are coalesced into a vortex. Figure 1c shows the start-up process of a nozzle with half-angle 30°. The starting wave has roughly a cylindrical shape, and the reflected wave, which is convex toward the nozzle axis, has two knee points near the walls, due to degeneracy of the separated flow. The whole system of waves moves downstream from the nozzle. Such configurations are usually observed [9, 10] when the whole system of waves is carried completely out of the nozzle and unseparated flow is established in the nozzle.

However, we observed that flows of this type, with overexpansion in steady-state nozzles, can create start-up conditions where the conical waves arising on interaction of the reflected wave with the boundary layer merge on the axis and form jets in the nozzle.

The process of forming jets occurs by intersection of the oblique shocks and by secondary formation of the transformed wave. Here periodic variations in the speed of the transformed wave are observed. The two-dimensional flow decomposes. Figure 2 shows photographs of start-up of a nozzle with half-opening angle of 30° in two different regimes: a) A transformed wave in the nozzle interacts with the boundary layer and the whole system moves downstream along the nozzle; b) the oblique shocks intersect, and a jet is formed within the nozzle; the test parameters are the following: the gas is nitrogen, p_0 = 30 torr, and M_0 = 2.3 (a) and 2.26 (b).

When the critical section is reduced one can obtain a periodic structure inside the nozzle, as is shown in Fig. 3. Here a photograph of start-up of a nozzle is shown for nitrogen with p_0 = 150 torr, M_0 = 1.8, and one sees both the starting wave, which has already passed into the constant area channel and has experienced Mach reflections at the channel inlet, and the reflected wave. One can see a jet and part of the transformed wave marked by an arrow.

From the time-dependent data obtained in the experiments one can obtain the x, t coordinates of all the waves. In order to compare the results with different theories one should first determine the gas parameters behind the reflected wave. On the sweeps we can measure the velocity of the reflected wave, and from the conservation laws one can determine the parameters of the gas behind it. It is assumed that behind the reflected wave the nitrogen is in a state of complete thermodynamic equilibrium. This state was determined from the speed of the reflected wave using tables in [13]. To determine the initial speed of the starting wave at the throat we can use a method of calculation employing decomposition of an arbitrary discontinuity [8]. Here one assumes that the incident wave decomposes after reflection into the starting wave at the throat and into the reflected wave, and behind the reflected wave the gas accelerates in a steady-state manner up to the critical parameters at the throat. As was shown in [5], the expansion wave has no appreciable influence on the distribution of the parameters ahead of the nozzle, and therefore for simplicity one can use the Alpher—White method in calculating the initial speed of the starting wave [14]. Then, to determine the speed of the passing wave one can use the Chisnell method [12] or the method of decomposition

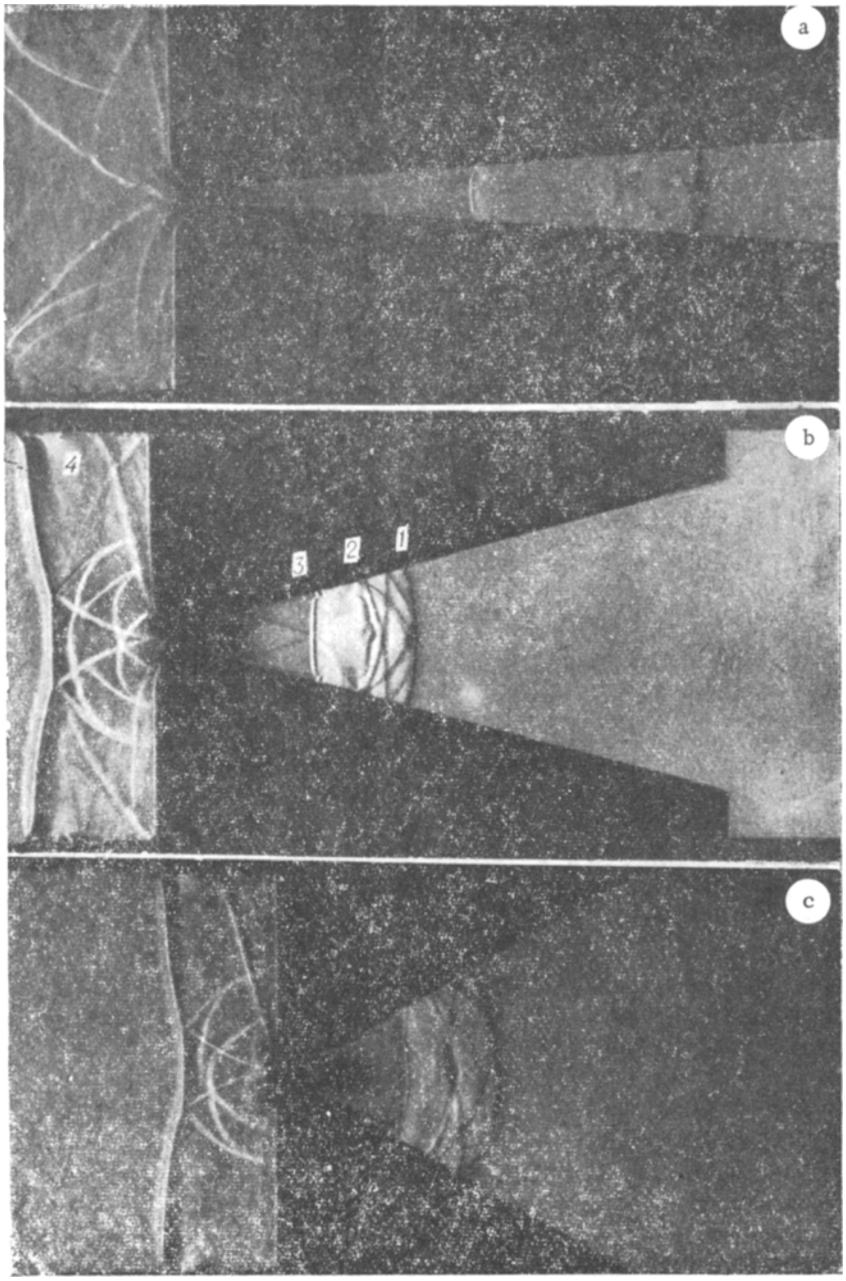

Fig. 1

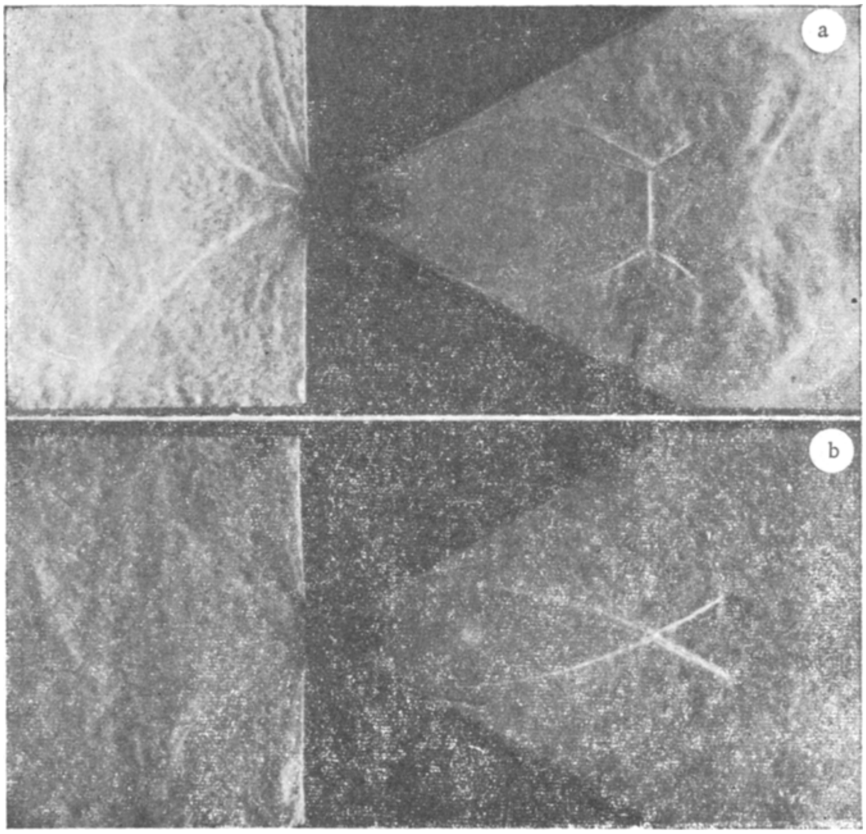

Fig. 2

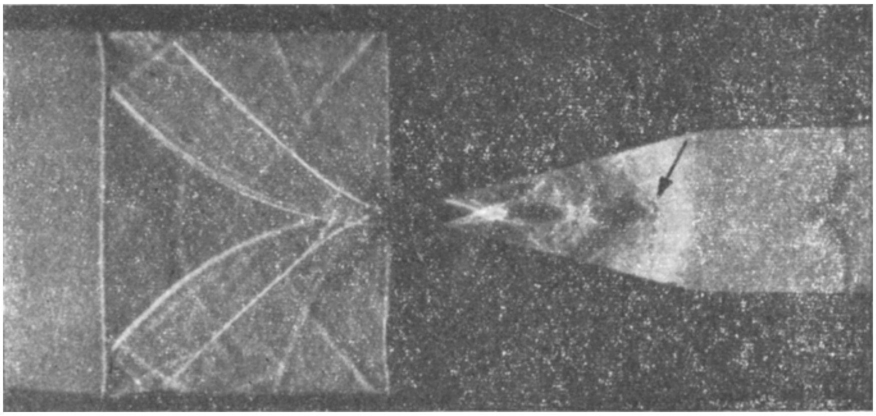

Fig. 3

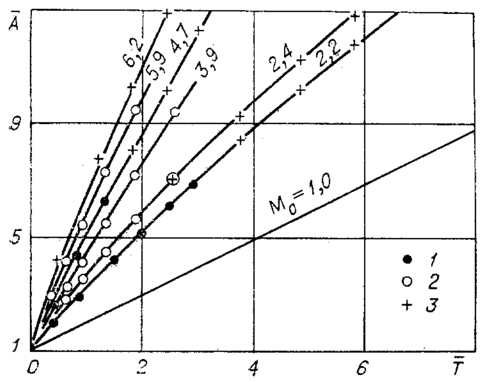

Fig. 4

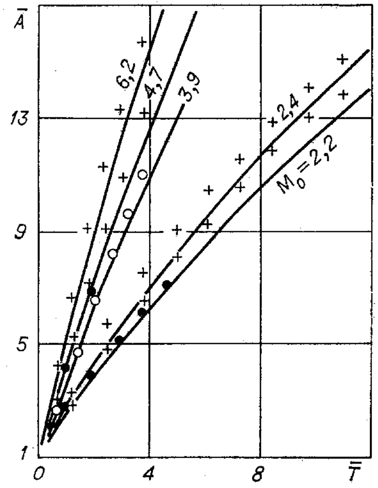

Fig. 5

of an arbitrary discontinuity [8]. In the Chisnell method we introduce some changes, since it is a matter of extending the analysis of [15] to the case of a finite change of section by integration of the individual elements. The Chisnell relation has the form $A^k(M^2 - 1) =$ const, where A is the cross-sectional area of the stream tube; M is the wave Mach number; k is a complex function of the wave intensity and the nature of the gas; for a strong shock wave in diatomic gases, $k \approx 0.4$.

It is clear that in this relation it is more reasonable to take not the cross-sectional area of the expanding channel, but the area of the attenuated shock itself. The difference between these quantities is not great for nozzles with small half-opening angles ($\alpha_0 = 5°$) and is small for moderate angles ($\alpha_0 = 15°$). This explains the good agreement with experiment of the operating calculations [10], where the starting wave was considered to be planar and the nozzle section was calculated from the formula

$$H = h + r(1 - \cos\alpha_0) + (x - x_2)\operatorname{tg}\alpha_0,$$

where x is the distance from a given section along the nozzle axis to the critical section, and x_2 is the distance to the point where the curved throat meets the straight line of the expanding part. As can be seen from Fig. 1, the shape of the starting wave is not planar but closer to cylindrical. Therefore for the Chisnell-method calculations the following expression was used for the shock area: $A = 2h + 2\alpha_0 x$, which is strictly valid for weak shock waves near sound waves. The use of this expression is particularly convenient for large nozzle half-angles ($\alpha_0 \sim 30°$), where the shock is really curved and the difference between A and 2H becomes noticeable. In addition this form of the expression for the nozzle cross section allows the theoretical and experimental data to be generalized very simply for different nozzle half-angles. In fact, the starting-wave trajectory can be described by the expression

$$t = \frac{1}{a_0}\int_0^x \frac{dx}{M(M_*, A/A_*)},$$

where a_0 is the initial sound speed; and M_* and A_* are the Mach number and the shock wave area at the critical section, respectively.

After a transformation we obtain

$$\frac{\alpha_0 a_0 t}{h} = \int_1^{\bar{A}} \frac{d\bar{A}}{M(M_*, \bar{A})} = \bar{T}.$$

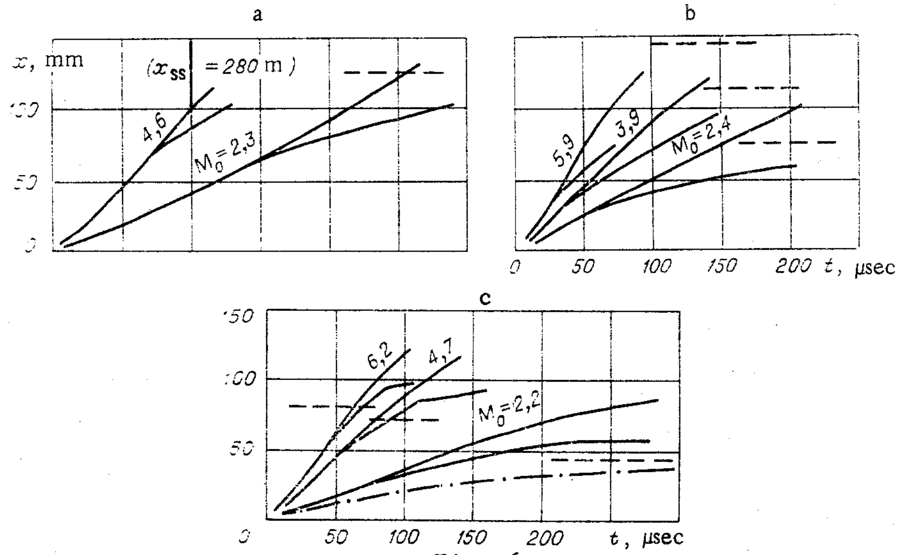

Fig. 6

Thus, the quantities $\overline{A} = A/A_* = 1 + \alpha_0 x/h$ and $\overline{T} = \alpha_0 a_0 t/h$ are convenient dimensionless coordinates for reducing the experimental data. The results of this reduction, obtained from the photographic records for various nozzles, and the theoretical calculations are shown in Fig. 4, where 1 shows the data for a half-opening angle of $\alpha_0 = 5°$, 2 shows data for $\alpha_0 = 15°$ (model 3), and 3 shows data for $\alpha_0 = 30°$; one can see very good agreement between these calculations and the experimental points.

Using the convenience of dimensionless coordinates we can postulate an approximation for calculating the starting wave in a reflected nozzle:

$$\overline{A} = 1 + M_0^{0.94}\overline{T}^{0.96(M_0-1)},$$

which is valid in the range of parameters $M_0 \sim 1$-7, $p_0 \sim 10$-30 torr, $\alpha_0 \sim 5$-30°, $\overline{A} \leqslant 16$.

The trajectories of the contact surface and the transformed wave can be calculated from the so-called asymptotic steady-state theory. This theory allows one to calculate the velocity of a system of waves resulting from decay of the primary shock wave [8]. Here we assume that the pressure and the velocity behind the initial shock are equal to the pressure and velocity behind the transformed wave, and that there is a steady flow expansion in the nozzle between the critical section and the transformed wave. We can vary the method somewhat for calculating the contact surface using this theory as follows. We will calculate the starting wave velocity by the Chisnell method described above, and retain the postulates of the theory that the expansion wave in the nozzle is steady and that the pressure is constant in the region between the forward and transformed waves. The results of comparing experiment and theory are shown in Fig. 5. As expected, the experimental points lie above the calculated curves, since the pressure actually falls somewhat behind the starting wave [10].

The experimental results can be approximated by the function

$$\overline{A} = 1 + 1.6(M_0 - 1)^{0.7}\overline{T}^{0.8}.$$

In regard to the transformed wave, as has been pointed out by other authors [9, 10], a calculation using steady-state theory gives a greatly overestimated value of the transformed wave strength. The actual velocity of the transformed wave lies between the steady-state calculation value and the speed of sound relative to the flow velocity in the steady-state gas flow in the nozzle. In addition, because of interaction with the boundary layer, the shape of the transformed wave may be planar, concave, or convex. Therefore, one cannot express the trajectory of motion in any generalized coordinates and assume a convenient approximation for the experimental data. Figure 6 shows the experimentally obtained transformed wave trajectories in the nozzle with various half-opening angles: a) $\alpha_0 = 5°$, b) $\alpha_0 = 15°$ (model 3), c) $\alpha_0 = 30°$ in x, t coordinates, where x is the coordinate along the nozzle axis reckoned from the throat, and t is the time in microseconds from the time of approach of the incident wave to the throat. The initial pressure is 10-30 torr and the gas is nitrogen. The bifurcation of the trajectory corresponds to the appearance in the field of view of the point of intersection of the oblique shocks (see Fig. 2b). The trajectory of the point of intersection of the oblique shocks is the lower line, while the upper line is the trajectory of the transformed wave. It can be seen from Fig. 6c that the trajectory of the point of intersection of the oblique shocks has a knee and becomes parallel to the t axis for small Mach numbers; i.e., a jet is generated in the nozzle.

It is well known that the steady-state theory predicts an increase in the transformed wave intensity and the possibility that it comes to rest at some section. For the tests conducted we used steady-state theory to calculate the transformed wave velocities and the nozzle sections (see Fig. 6, dashed lines) where the transformed wave should stop. In Fig. 6c, for the condition $M_0 = 2.22$, a dashed-dot line shows the theoretical trajectory of the transformed wave. As expected from analysis of all the references, the actual trajectory lies above this curve. This sort of law exists for all the other conditions. The main cause for the difference between theory and experiment (according to the data of [10]) lies in breakdown of one of the two postulates of steady-state theory, the postulate that the parameters are constant between the starting wave and the transformed wave. The experimental data confirm this view.

Thus, it would appear that the actual time for the nozzle to start is less than the calculated value, judging from the passage of the transformed wave past a given section. However, because of interaction of the transformed wave with the boundary layer there is flow separation and oblique waves are formed. In estimating the start-up time one should follow the time for formation of the points of intersection of the inclined wave and its trajectory. When the jet forms the nozzle beccmes self-blocked.

It can be seen from Fig. 6 that for nozzles with large half-opening angle the theory predicts better conditions for nozzle start-up than occur in practice. In reality, at sections of the nozzle with a half-opening angle of 30°, where steady-state theory predicts that the transformed wave will stop, the actual transformed wave is weaker and is carried through this section by the flow. The blocking of the nozzle occurs at the large cross section because of intersection of oblique waves arising upon interaction of the transformed wave with the boundary layer. This situation is different in nozzles with half-opening angles of 5 and 15° (see Fig. 6a, b). Here the actual conditions are worse in comparison with theory. The flow becomes inhomogeneous and the nozzle blocks earlier than is predicted by steady-state theory.

The jet created corresponds to a steady-state jet in nozzles operating with overexpansion, i.e., in conditions where the gas in a nozzle expands to smaller pressures than the initial pressure at the nozzle exit. The jet formed at the moment of start-up is not steady-state in the sense that it is determined by the ratio of the initial pressure in the nozzle and the parameters behind the reflected wave. The parameters determining the position of the jet in the nozzle depend on the channel geometry below the nozzle exit. For example, if the nozzle has a transition in a channel of constant cross section, as shown in Fig. 3, then after Mach reflection the starting wave moves along the channel at constant velocity. The pressure behind the starting wave will also in fact determine the counterpressure for this stage of the start-up. The jet will be quasisteady as long as the conditions do nct vary behind the starting wave. If there is an expansion beyond the nozzle exit, the starting wave will decelerate, the pressure behind it will fall, and the quasisteady jet will adjust to the new counterpressure. When the starting wave degenerates into a sound wave, the flow in the nozzle will undergo transition to a steady-state condition.

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EXPERIMENTAL INVESTIGATION OF TOTAL FLOW AND DIRECTIONAL DIAGRAMS IN DISCHARGE OF GAS TO VACUUM THROUGH CAPILLARIES OF DIFFERENT LENGTHS

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An experimental investigation of discharge of gas into a vacuum through cylindrical channels over a wide range of source gas pressure is of interest, since this phenomenon finds wide application in both engineering and scientific research [1, 2]. In discharge of gas from a channel into vacuum different flow conditions are found, from viscous to free-molecular, and this makes the problem very different from the well-studied problem where gas flows with negligibly small pressure differences in comparison with the average pressure, and complicates theoretical study.

The flow of gas through cylindrical capillaries with different ratios of length to diameter l/d has been studied by various authors (e.g., [3-7]). In [3, 4] an unsteady flow method was used, in which the pressure drop in the channel is considerably less than the average pressure. A large pressure drop in the channel was allowed for in [5]. However, in that paper the author restricted his study to an intermediate flow regime. To describe the observed results, as a rule, one uses either semiempirical formulas [3, 4], or quite rigorous theoretical formulas, obtained for a limiting viscous flow regime [6]. At present there are no theoretical papers describing gas discharge into vacuum through a cylindrical channel under arbitrary flow conditions.

The directional diagrams (the angular distribution of the molecular beam intensity) have been investigated experimentally by various authors (e.g., [8-10]). A rather complete study of molecular beams was conducted in [11], where the directional diagrams were measured for an ammonia beam over a wide pressure range and for channels of different lengths. The results obtained agree satisfactorily with theoretical calculations made in [12]. Later, the problem of generating molecular beams was studied theoretically in [13].

The present paper presents results of a systematic experimental study of the influence of geometric dimensions of circular capillaries and the nature of the gas on the total molecular flow and the shape of the directional diagrams of molecular beams of hydrogen and carbon dioxide. The results obtained have been compared with available theoretical data.

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